Fundamentals of Communications Engineering

Department of Communications Engineering, College of Engineering, University of Diyala, 2016-2017

Class: Second Year
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Room: Comm-02

Solutions to Homework 01

Lecture 1 Exercises solutions

1 Ans. $f(t) = t^3 + t + t^2$

$$f(-t) = (-t)^3 + (-t) + (-t)^2$$
$$= -t^3 - t + t^2$$

$$f_e(t) = \frac{1}{2}\left[f(t) + f(-t)\right]$$

$$\therefore f_e(t) = \frac{1}{2}\left[t^3 + t + t^2 - t^3 - t + t^2\right]$$

$$f_e(t) = \frac{1}{2}\left[2t^2\right] = t^2$$

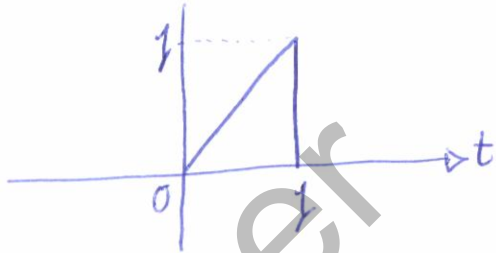
$$\therefore \boxed{f_e(t) = t^2}$$

$$f_o(t) = \frac{1}{2}\left[f(t) - f(-t)\right] = \frac{1}{2}\left[t^3 + t + t^2 + t^3 + t - t^2\right]$$

$$f_o(t) = \frac{1}{2}\left[2t^3 + 2t\right]$$

$$\boxed{f_o(t) = t^3 + t}$$

(2) $f(t) = \begin{cases} t & 0 \leq t < 1 \\ 0 & \text{elsewhere} \end{cases}$

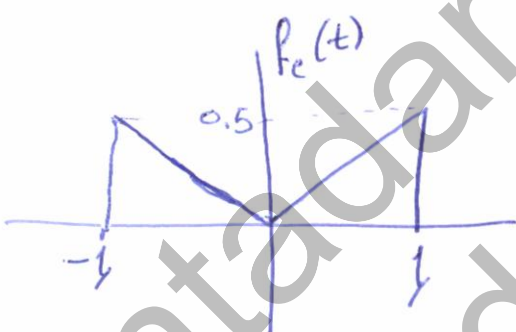


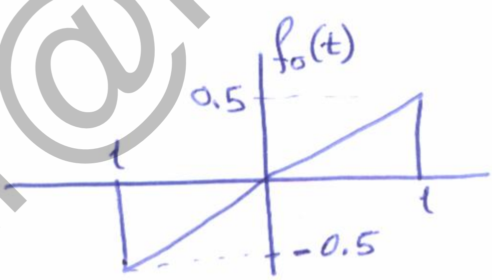
Solution

$$f(-t) = \begin{cases} -t & -1 < t \leq 0 \\ 0 & \text{elsewhere} \end{cases}$$

$$f_e(t) = \frac{1}{2}\left[f(t) + f(-t)\right] = \begin{cases} \frac{1}{2}t & 0 \leq t < 1 \\ -\frac{1}{2}t & -1 \leq t < 0 \\ 0 & \text{elsewhere} \end{cases}$$

$$f_o(t) = \frac{1}{2}\left[f(t) - f(-t)\right] = \begin{cases} \frac{1}{2}t & 0 \leq t < 1 \\ \frac{1}{2}t & -1 \leq t < 0 \\ 0 & \text{elsewhere} \end{cases}$$





3 $f(t) = 2t^4 - 5t^3 + 2t^2 + t - 4$

Solution

$$f_e(t) = \frac{1}{2}\left[f(t) + f(-t)\right]$$

$$f(-t) = 2(-t)^4 - 5(-t)^3 + 2(-t)^2 + (-t) - 4$$

$$= 2t^4 + 5t^3 + 2t^2 - t - 4$$

$$\begin{array}{rl} f(t) + f(-t) = & 2t^4 - 5t^3 + 2t^2 + t - 4 \\ + & 2t^4 + 5t^3 + 2t^2 - t - 4 \\ \hline f(t) + f(-t) = & 4t^4 \qquad + 4t^2 \qquad - 8 \end{array}$$

$$f_e(t) = \frac{1}{2}\left[4t^4 + 4t^2 - 8\right] = 2t^4 + 2t^2 - 4$$

$$f_o(t) = \frac{1}{2}\left[f(t) - f(-t)\right]$$

$$\begin{array}{r|l} f(t) & 2t^4 - 5t^3 + 2t^2 + t - 4 \\ - f(-t) & -2t^4 - 5t^3 - 2t^2 + t + 4 \\ \hline & -10t^3 + 2t \end{array}$$

$$\therefore f_o(t) = \frac{1}{2}\left[-10t^3 + 2t\right] = -5t^3 + t$$

4) $g(t) = \dfrac{1}{t-1}$

solution

$$g_e(t) = \frac{1}{2}\left[g(t) + g(-t)\right]$$

$$g_o(t) = \frac{1}{2}\left[g(t) - g(-t)\right]$$

we need $g(-t)$

$$g(-t) = \frac{1}{-t-1}$$

$$g_e(t) = \frac{1}{2}\left[\frac{1}{t-1} + \frac{1}{-t-1}\right] = \frac{1}{2}\left[\frac{-t-1+t-1}{-(t+1)(t-1)}\right]$$

$$g_e(t) = \frac{1}{2}\left[\frac{-2}{-(t^2-1)}\right] = \frac{1}{t^2-1}$$

$$g_o(t) = \frac{1}{2}\left[\frac{1}{t-1} - \frac{1}{-t-1}\right] = \frac{1}{2}\left[\frac{-t-1-t+1}{-(t^2-1)}\right] = \frac{1}{2}\left[\frac{-2t}{-(t^2-1)}\right]$$

$$\therefore g_o(t) = \frac{t}{t^2-1}$$

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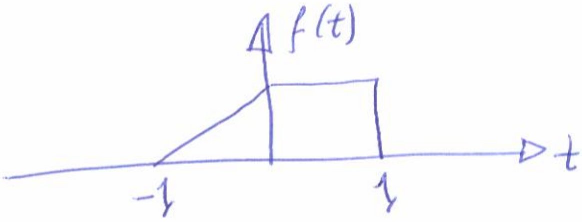
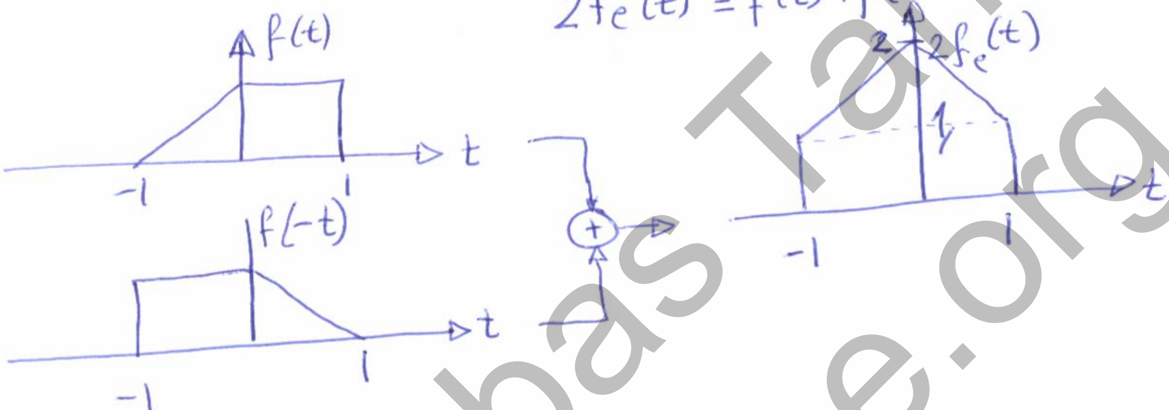


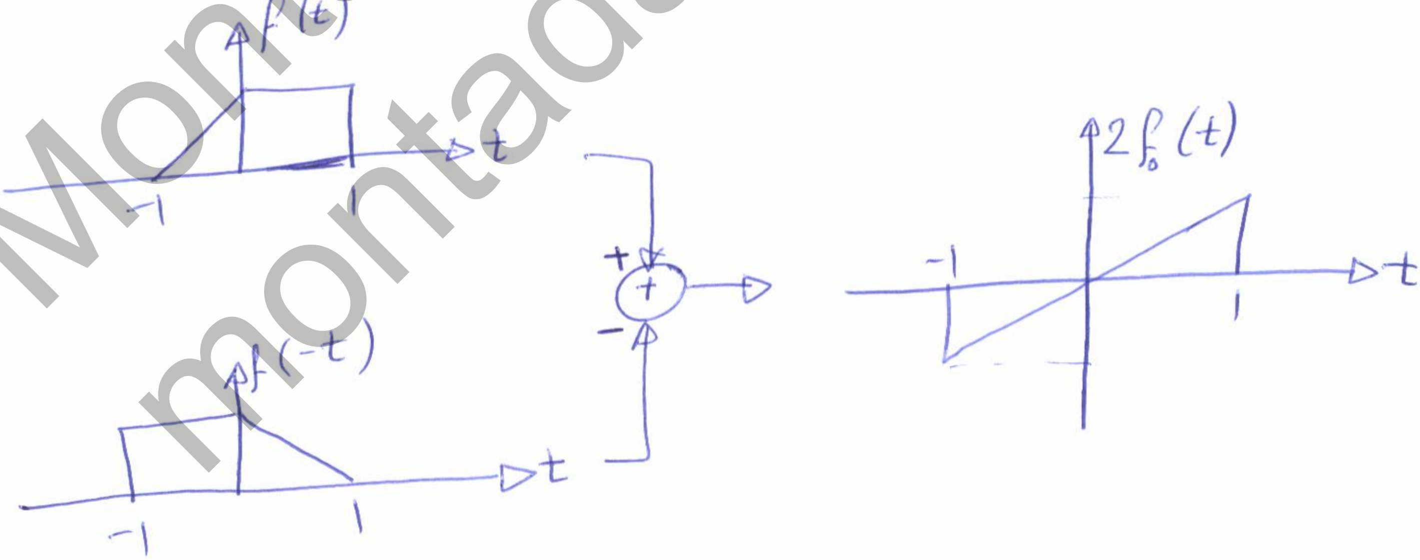
Solution even component is: $f_e(t) = \frac{1}{2}\left[f(t) + f(-t)\right]$

$$2f_e(t) = f(t) + f(-t)$$



odd component: $f_o(t) = \frac{1}{2}\left[f(t) - f(-t)\right]$

$$2f_o(t) = f(t) - f(-t)$$



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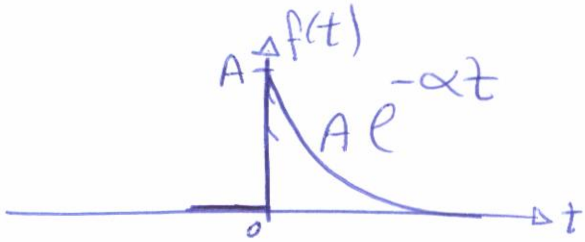


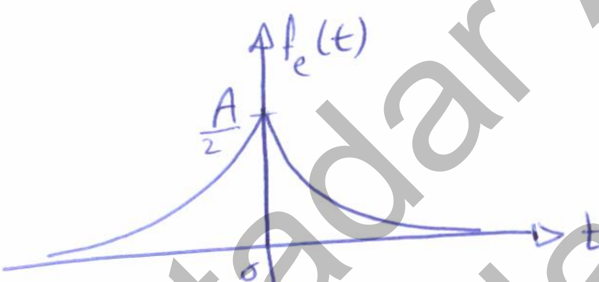
this figure tells that the function is

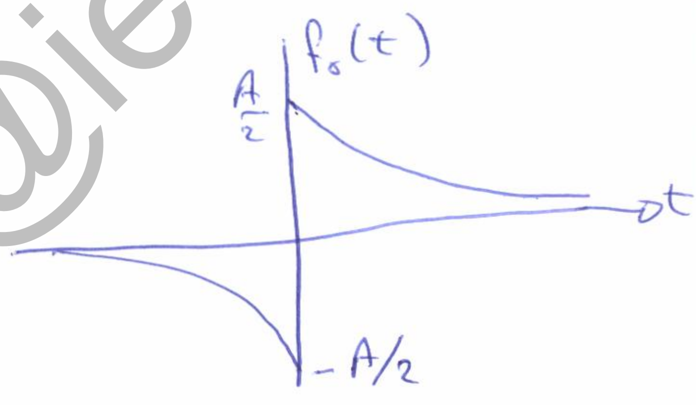
$f(t) = Ae^{-\alpha t}$ $\quad 0 \leq t \leq \infty$

$f(-t) = Ae^{\alpha t}$ $\quad -\infty \leq t \leq 0$

$f_e(t) = \frac{1}{2}[f(t) + f(-t)] = \frac{1}{2}[Ae^{-\alpha t} + Ae^{\alpha t}]$

$f_o(t) = \frac{1}{2}[f(t) - f(-t)] = \frac{1}{2}[Ae^{\alpha t} - Ae^{-\alpha t}]$





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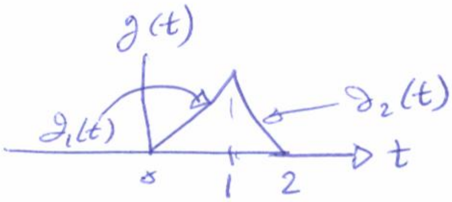


$$g_1(t) = at + b$$

$$g_1(t) = 0,\ t = 0 \Rightarrow \boxed{0 = b}$$

$$g_1(t) = 1,\ t = 1 \Rightarrow 1 = a + 0 \rightarrow \boxed{a = 1}$$

$$\therefore \boxed{g_1(t) = t \qquad 0 \leq t \leq 1}$$

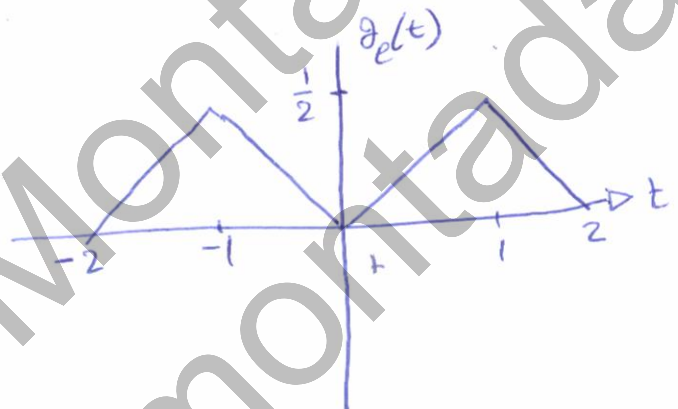
$$g_2(t) = 1,\ t = 1 \Rightarrow 1 = a + b \Rightarrow \boxed{b = 1 - a}$$

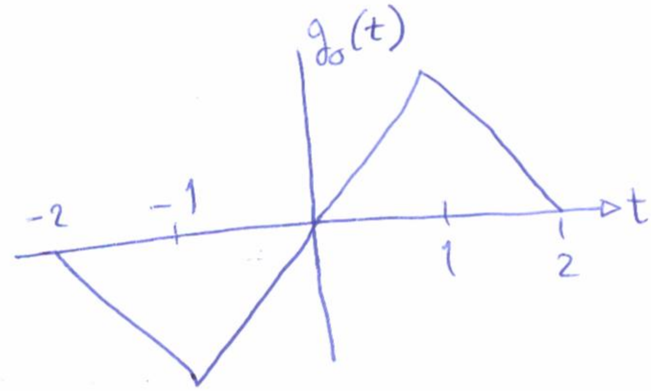
$$g_2(t) = 0,\ t = 2 \Rightarrow 0 = 2a + b \Rightarrow 0 = 2a + 1 - a \Rightarrow \boxed{a = -1}$$

$$\boxed{b = 2}$$

$$\therefore \boxed{g_2(t) = -t + 2 \qquad 1 \leq t \leq 2}$$

$$g(t) = \begin{cases} t & 0 \leq t \leq 1 \\ -t + 2 & 1 \leq t \leq 2 \\ 0 & \text{elsewhere} \end{cases} \qquad g(-t) = \begin{cases} -t & -1 \geq t \leq 0 \\ t + 2 & -2 \geq t \leq -1 \\ 0 & \text{elsewhere} \end{cases}$$





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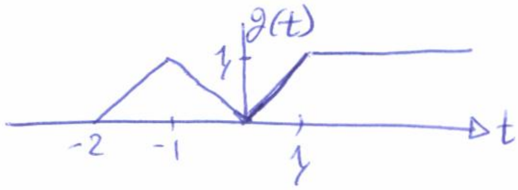
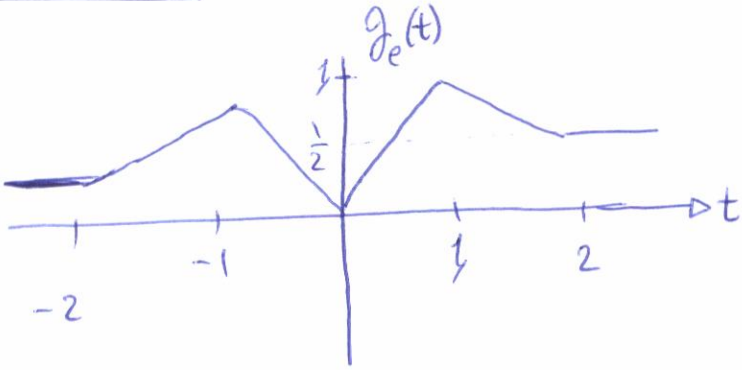
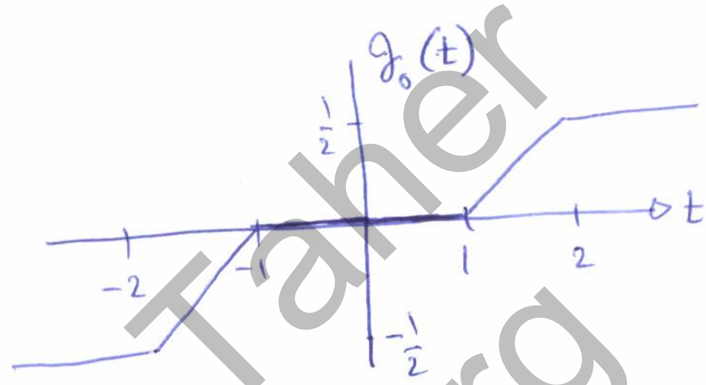


Solution





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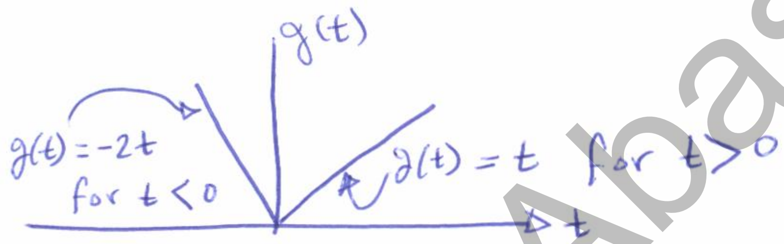
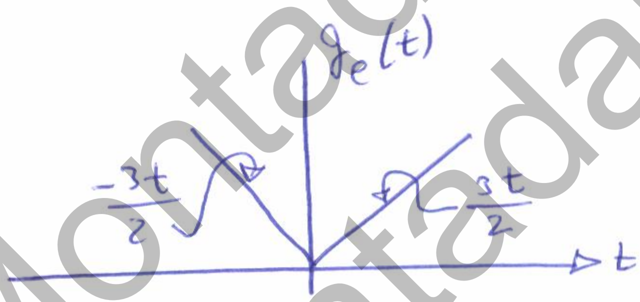
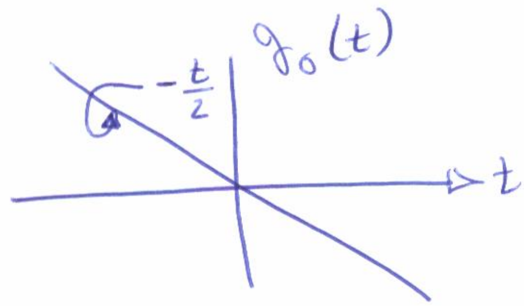


Solution





10 $h(t) = \cos(t) + \sin(t) + \sin(t)\cos(t)$.

solution

$$h(-t) = \cos(-t) + \sin(-t) + \sin(-t)\cos(-t)$$

$$h(-t) = \cos(t) - \sin(t) - \sin(t)\cos(t)$$

$$h_e(t) = \frac{1}{2}\left[h(t) + h(-t)\right] = \frac{1}{2}\left[\cos(t) + \cancel{\sin(t)} + \cancel{\sin(t)\cos(t)} + \cos(t) - \cancel{\sin(t)} - \cancel{\sin(t)\cos(t)}\right]$$

$$h_e(t) = \frac{1}{2}\left[2\cos(t)\right] = \cos(t) \quad \blacksquare$$

$$h_o(t) = \frac{1}{2}\left[h(t) - h(-t)\right] = \frac{1}{2}\left[\cancel{\cos(t)} + \sin(t) + \sin(t)\cos(t) - \cancel{\cos(t)} + \sin(t) + \sin(t)\cos(t)\right]$$

$$h_o(t) = \frac{1}{2}\left[2\sin(t) + 2\sin(t)\cos(t)\right]$$

$$h_o(t) = \sin(t) + \sin(t)\cos(t)$$

11 $y(t) = 1 + t\cos(t) + t^2 \sin(t) + t^3 \sin(t)\cos(t)$.

solution

* we know that even × even = even
  even × odd = odd
  odd × even = odd
  odd × odd = even

* since t is odd and $\cos(t)$ is even, then $t\cos(t)$ is odd.

* since t^2 is even and $\sin(t)$ is odd, then $t^2\sin(t)$ is odd.

* since t^3 is odd, $\sin(t)$ is odd, and $\cos(t)$ is even, then $t^3\sin(t)\cos(t)$ is even.

$\therefore$ $y_e(t) = 1 + t^3 \sin(t)\cos(t)$

$y_o(t) = t\cos(t) + t^2 \sin(t)$

12 $f(t) = \left|\sin\left(\frac{-5\pi t}{8} + \frac{\pi}{2}\right)\right|$, $\boxed{\text{it is periodic}}$

Solution $f(t) = \left|\sin\left(\frac{-5\pi t}{8} + \frac{\pi}{2}\right)\right| = \left|\cos\left(\frac{5\pi t}{8}\right)\right|$

$$f(t+T_0) = \left|\cos\left(\frac{5\pi t}{8} + \frac{5\pi T_0}{8}\right)\right|$$

if $\frac{5\pi T_0}{8} = \pi$, $f(t)$ will repeats itself, thus,

$$\boxed{T_0 = \frac{8}{5}}$$

13 $f(t) = \sin\left(\frac{6\pi t}{7}\right) + 2\cos\left(\frac{3t}{5}\right)$

* $\sin\left(\frac{6\pi t}{7}\right)$ is periodic (alone)

* $\cos\left(\frac{3t}{5}\right)$ is periodic (alone)

Since we know that $\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0}$

$\sin(\omega_1 t) \Rightarrow \omega_1 = \frac{6\pi}{7} = \frac{2\pi}{T_1} \Rightarrow \boxed{T_1 = \frac{7}{3}}$ for the first term

$\cos\left(\frac{3t}{5}\right) \Rightarrow \omega_2 = \frac{3}{5} = \frac{2\pi}{T_2} \Rightarrow \boxed{T_2 = \frac{10\pi}{3}}$

$\frac{T_1}{T_2} = \frac{7/3}{\frac{10\pi}{3}} = \frac{7}{3}\,\frac{3}{10\pi} = \frac{7}{10\pi}$ ⟸ this is not a rational number

$\therefore$ $f(t)$ is nonperiodic

(14) $f(t) = e^{\frac{j3\pi t}{8}} + e^{\frac{\pi t}{86}}$

Solution $e^{j\frac{3\pi t}{8}}$ is periodic with period $T_1 = \frac{2\pi}{\frac{3\pi}{8}} = \frac{16}{3}$

$e^{\frac{\pi t}{86}}$ is not-periodic

since the second term is-not periodic, then the signal is not-periodic.

(15) $r(t) = 7 \underbrace{\sin(2\pi t)}_{\text{odd}} \underbrace{(3 + \underbrace{\cos(2\pi 2t)}_{\text{even}})}_{\text{even}}$

$\therefore$ r(t) is odd.

(16) $c(t) = \underbrace{41 t^2}_{\text{even}} + \underbrace{\cos(7t)}_{\text{even}}$

$\therefore$ c(t) is an even function

(17) $h(t) = \sin(3\pi t)\cos(\pi t)$

$* \sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$

$+ \sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$

$\sin(\alpha)\cos(\beta) = \frac{1}{2}\left[\sin(\alpha + \beta) + \sin(\alpha - \beta)\right]$

$\alpha = 3\pi t$

$\beta = \pi t$

$\therefore h(t) = \sin(3\pi t)\cos(\pi t)$

$h(t) = \frac{1}{2}\sin(4\pi t) + \frac{1}{2}\sin(2\pi t)$

$\sin(4\pi t)$ } periodic $T_1 = \frac{1}{2}$

$\sin(2\pi t)$ } periodic $T_2 = 1$

$\frac{T_1}{T_2} = \frac{\frac{1}{2}}{1} = \frac{1}{2}$

$\therefore$ h(t) is periodic

18 $g(t) = \begin{cases} \cos(10\pi t) & -12 \leq t \leq 12 \\ 0 & \text{elsewhere} \end{cases}$

Solution

$g(t)$ is non-periodic since it is defined over the range $-12 \leq t \leq 12$ only.

(19) solution since the signal $r(t)$ non-periodic, hence it is an energy signal

$$\mathcal{E}_r = \int_{-\infty}^{\infty} |r(t)|^2 dt = \int_{-3}^{3} 9\, dt = 54$$

(20) solution the signal is aperiodic, then $x(t)$ is contains energy.

$$\mathcal{E}_x = \int_{-1}^{1} |4t^3|^2 dt = 16 \int_{-1}^{1} t^6 dt = \frac{16}{7} t^7 \Big|_{-1}^{1} = \frac{16}{7}[1+1] = \frac{32}{7}$$

(21) solution this signal is periodic with fundamental period $T = 8$

$$h(t) = \begin{cases} 5 & -2 \leq t \leq 2 \\ 0 & 2 < |t| \leq 4 \end{cases}$$

$$P_z = \frac{1}{8} \int_{-2}^{2} 25\, dt = \frac{25}{8}[2+2] = 12.5$$